UNIVERSITY AT ALBANY
State University of New York

College of Arts and Sciences
Department of Mathematics and Statistics

Algebra/Topology Seminar

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Homotopy Groups of Gauge Groups

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1:15 p.m. in ES-143

Abstract. Let A be a C^*-algebra. Its unitary group, UA, contains a wealth of topological information about A. However, the homotopy type of UA is unknown even for $A = M_2(\mathbb{C})$. There are various simplifications which have been considered. The first, well-traveled road, is to pass to $\pi_*(U(A \otimes \mathcal{K}))$ which is isomorphic (with a degree shift) to $K_*(A)$. This approach has led to spectacular success in many arenas, as is well-known.

A different approach is to consider $\pi_*(UA) \otimes \mathbb{Q}$, the rational homotopy of UA, or, to be brave/reckless, to consider $\pi_*(UA)$ itself. We report on progress in the calculation of these functors for A an algebra of sections of a locally trivial bundle of C^*-algebras over a compact metric space X with C^*-algebra fibre B, so that UA is the associated gauge group. If the bundle is trivial then $UA \cong F(X, UB)$ and the Federer spectral sequence (as generalized to compact metric spaces) may be used. Our interest is the case where the bundle is non-trivial, so that A is a twisted algebra. We construct a spectral sequence converging to the homotopy of the gauge group $\pi_*(UA)$ with $E_2 \cong H^*(X; \pi_*(UB))$ and a similar spectral sequence converging to $K_*(A)$.

In the case $X = S^k$ we produce a Wang sequence relating the homotopy of the gauge group UA and of UB and explain a conjecture identifying the differential in the gauge group sequence in terms of the classifying map of the bundle and a Samelson product.

These results are joint work and work in progress with J. Klein, G. Lupton, N.C. Phillips, S. Smith, and E. Dror-Farjoun.