Abstract. The “virtual conjectures” in low-dimensional topology, stated by Thurston in 1982, postulated that every hyperbolic 3-manifold has finite covers that are Haken and fibered, with large Betti numbers. These conjectures were resolved in 2012 by Agol and Wise, using the machine of special cube complexes. Since that time, many mathematicians have asked how big a cover one needs to take to ensure one of these desired properties.

We begin to give a quantitative answer to this question, in the setting of alternating links in $S^3$. If an alternating link $L$ has a diagram with $n$ crossings, we prove that the complement of $L$ has a special cover of degree less than $72((n - 1)!)^2$. As a corollary, we bound the degree of the cover required to get Betti number at least $k$. We also quantify residual finiteness, bounding the degree of a cover where a closed curve of length $k$ fails to lift. This is joint work with David Futer.